"OVIDIUS" UNIVERSITY OF CONSTANŢA
DOCTORAL SCHOOL OF MATHEMATICS
DOMAIN - MATHEMATICS

Summary
Ph.D THESIS

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Summary
MATHEMATICAL MODELING OF CRACK PROPAGATION IN PIEZOELECTRIC MATERIALS WITH INITIAL FIELDS

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Contents

Chapter 1

Introduction

Short history

Piezoelectric materials are widely used in medical imaging, acoustics, in the manufacture of chips, sensors, hydrophones, devices of actuation of the intelligent components of the energy collectors. The properties mechanical properties of the composite materials coupled with the electrical properties of the materials piezoelectric and sometimes also with magnetic properties offers the purpose of being created intelligent structures capable of responding to internal and surrounding changes, being used due to their ability to reduce the concentration of the state of tension and the increase resistance to breakage in the field of electronic instruments, devices with microwaves and in optoelectronics.

The study of the stress field in an infinite anisotropic or piezoelectric body with an elliptical hole has been and still is an interesting and challenging problem among researchers in Solid Mechanics. Due to its simplicity and importance, these studies have been given considerable interest in the last decades by many authors. The study of a crack or an elliptical inclusion in composite materials is of great importance in theory and applications and has been carried out in many interesting works, e.g. ([10], [17], [18], [53], [70]). Due to the anisotropy of the material and the effects of electroelastic coupling, the mathematical modeling of piezoelectric materials with defects such as cracks, inclusions, weakening is not simple to develop from a mathematical point of view, examples in this sense being the works ([9], [29], [46], [66], [67], [68], [78], [76]).

Using the Lekhnitskii formalism ([14], [42] [50]) for anisotropic elastic composite materials and Guz's method ([28]) for prestressed elastic composite materials, Soos ([4], [5], [19], [64]) obtain the representation by two full potentials $\Psi_j = \Psi_j(z_j),\ j = 1.2$ in the case of the incremental antiplane problem of the incremental stress components

$\boldsymbol{\theta}$, and of the electrical incremental displacement $\boldsymbol{\Delta}$.

Applications of piezoelectric materials are of particular importance for the use of sensors and actuators in smart structures in medical imaging, in ultrasound applications, in acoustics, and in many other fields of engineering.

In the last decades, a lot of researchers have conducted studies in the field of mathematical modeling of piezoelectric materials of important coupling effects between electric and mechanical fields. Eringen and Maugin in the first volume of the fundamental monograph ([24]) studied the stability of piezoelectric materials using the theory of small strains superimposed on large strains of static elastic fields. Sosa ([67]-[68]) studied crack propagation in two- and three-dimensional transversely isotropic piezoelectric media within the formalism of complex variable techniques. In one of the first papers on interface fracture of multiphase piezoelectric materials, Suo et al ([69]), developed a new theory of fracture mechanics to determine the fracture intensity and crack propagation in piezoelectric ceramic materials under hysteresis conditions. With the help of Muskhelishvili's or Lekhnitskii's formalism applied to cracked piezoelectric materials, many authors obtained important and imperative results for the study of propagation, Huang ([30]) or crack interactions, Crăciun ([15]-[20]). Bardzokas and collaborators ([7]-[9]) studied the failure of piezoceramic plates having defects in the form of cracks/inclusions and holes, reducing electroelasticity boundary value problems to solving systems of integral equations.

Crack problems at the interface of piezoelectric materials have received considerable attention from many researchers ([29], [57]). Recent contributions have been made in the specialized literature by other authors, ([46], [74]), who provided analytical solutions for interface cracking problems, in the case of impermeable or permeable interface cracks. The problem of mathematical modeling of the initially deformed piezoelectric material actuated by initial mechanical and electric fields containing a reinforced crack is insufficiently studied to date.

In recent years, many researchers have been attracted to structural studies of micro- and nanomaterials due to their special electronic, electrical, and mechanical properties. Thanks to these properties, nanomaterials are used as basic nanobar structural components in microelectromechanical systems (MEMS) or nanoelectromechanical systems (NEMS) as well as in piezoelectric devices. However, they have caused major problems as we move to the micro or nano scale. Therefore, future studies of piezoelectric materials are needed to increase the performance of the manufacturing technology of sensors, filters, actuators, transducers, etc.

Piezoelectric thin films are integrated with structural components to obtain MEMS or NEMS smart structure and with other composite materials used for other required functions. Piezoelectric crystals produce electrical charge when a certain mechanical force is applied and vice versa. Thus, the piezoelectric material helps us to actuate the structure while it performs piezoelectric, that is, the applied electric field generates the mechanical deformation.

MEMS/NEMS include the use of transducers for energy harvesting in mechanical engineering, navigation, aerospace and marine, medical ultrasound imaging, ink-jet printing, fluid monitoring using piezoelectric power supplies, and surface acoustic wave devices. With the advancements of modern MEMS/NEMS systems, the required power supplies are reduced and therefore we are oriented towards new amendments to the basic theory of piezoelectricity.

There are a large number of research papers describing the basic theories in the study of piezoelectric materials and other related materials, of which we will cite only a few in the following. Ieșean, ([39]) investigated and held fundamentals in the case of the plane deformation problem for homogeneous anisotropic piezoelectric materials. Liang and Shen, ([44]) examined Bernoulli-Euler bending of nanorods with piezoelectric effect. Eom and Trolier-McKinstry, ([23]) studied the design of thin film MEMS hetero-structure with piezoelectric effect. Vahdat and his collaborators, ([71]) investigated the resonator sandwich structure between two piezoelectric layers. Sadek and Abukhaled, ([56]) analyzed the vibrations of a beam due to sudden heat by a piezoelectric actuator. Kumar and Sharma, ([38]) studied the thermoelastic damping in a piezothermoelastic beam using the fractional order derivative thermal equation for a transversely isotropic material. Li and He, ([43]) studied piezoelectric bar with nonlocal elasticity, fractional order thermal conduction and mobile heat source. Zenkour, ([73]) studied the thermomechanical response of the microbar using the modified torque stress analysis theory subjected to two temperatures. And other researchers studied the behavior of the piezoelectric nanobar, such as Sharma and Kaur, ([61], [62]), Abouelregal and Zenkour, ([2]), Lata and Kaur, ([40]).

Purpose and objectives of the research

In this doctoral thesis, I propose to study and obtain the following results:

1. We have obtained a solution in an elementary, compact form for the complex potentials and the incremental stress and electric fields for the problem of an elliptical inclusion in a prestressed and prepolarized piezoelectric material acted at large distances by constant antiplane stresses and using conformal transformation.

2. In the case of a piezoelectric material initially acted upon by mechanical and electric fields and containing a reinforced crack of length $2a > 0$ located on the Ox_1 axis, subject to Mode III antiplane classical fracture, we also obtained the solution of the complex potential problems, the incremental displacement and tension fields, as

well as the propagation direction of a reinforced crack in a PZT-type piezoelectric material.

3. In the context of the generalized piezothermoelastic theory, new mathematical models for the piezothermoelastic type nanobars are obtained, thus determining the expressions for the lateral deflection, the electric potential, the thermal moment, the thermoelastic damping and the frequency shift.

The original results of the above three studies obtained by the author are presented each in Chapters 3, 4 and 5. Also, this doctoral thesis contains this introductory chapter, one presenting the preliminaries of the study, one chapter containing the conclusions of the doctoral thesis studies and one chapter presenting the bibliographic titles.

Acknowledgements

I would like to thank my PhD supervisor, prof. dr. Crăciun Eduard-Marius, who guided, encouraged and supported me throughout the duration of my doctoral studies, as well as professors from the guidance committee who helped me in my training during these years of preparation for my doctoral studies.

Chapter 2

Preliminaries

The original results of the doctoral thesis can be found in Chapters 3-5.

In Chapter 3 of this doctoral thesis I studied the problem of an elliptical inclusion/crack in a prestressed and prepolarized piezoelectric material loaded with antiplane shear stresses, constant and uniform, according to the third mode of fracture, using the representation of the incremental elastic and electric fields by two complex potentials $\Psi_j = \Psi_j(z_j), \ j = 1.2$ and the technique of conformal representation of elliptical crack regions outside the unit circle. The unknown coefficients of the analytic functions $\Psi_1(z_1)$ and $\Psi_2(z_2)$ represented by two Laurent series in two complex planes are determined from the boundary conditions. In the case of a pre-stressed and pre-polarized piezoelectric material of class $\bar{4}2m$ with an elliptical crack-type inclusion, when the minor semi-axis tends to zero, the complex potentials obtained in this chapter have the same form as those resulting from the crack propagation problem by solving the Riemann-Hilbert problem, ([19], [64]).

In Chapter 4 we analyzed the antiplane state of prestressed and prepolarized piezoelectric materials containing a bridged crack, following the bridge crack model used by Bigoni and co-workers ([12], [72]). Using the famous monograph of Eringen and Maugin ([24]) we derive the laws of incremental equilibrium, the constitutive equations and the sufficient conditions under which incremental antiplane states can exist in the prestressed and prepolarized piezoelectric material of PZT type ([63]), having the polarization axis of the material in the positive Ox_3 direction. In this case, the incremental antiplane state can be represented by two complex potentials and if the applied incremental voltage has a constant value, we determine the asymptotic expressions of the complex potentials and the asymptotic representations of the incremental mechanical and electrical fields. Extending the strain energy density criterion of Sih from the case of isotropic materials ([59], [63]) to the case of prestressed and prepolarized piezoelectric materials, we determine the direction of propagation of the bridged crack for a certain PZT piezoelectric material depending on the stiffness constant, and the initial elastic and electric fields.

In Chapter 5 we studied the behavior of a piezothermoelastic nanobar with fixed ends and kept at a constant temperature, using the theory of generalized piezothermoelasticity. From the mathematical model thus formulated, we obtain the dimensionless expressions for the lateral deviation, the electric potential, the thermal moment, the thermoelastic damping and the frequency shift. In the context of the generalized piezothermoelastic theory, using graphical representations with the aid of the MATLAB program, we studied the influence of the frequency effect in the representation of the solutions obtained for the lateral deviation, electric potential, thermal moment, thermoelastic damping and of the frequency shift as a function of the length of the nanobar within the framework of the coupled theory (CT), the Lord-Shulman theory (LS) and the Green-Lindsey (GL) theory, respectively.

Chapter 3

Prestressed and prepolarized piezoelectric material with an elliptical crack

The main goal of this chapter is to obtain new results regarding the solution expressed by complex potentials of the problem of an elliptical crack in a prestressed and prepolarized piezoelectric material acted by constant and uniform antiplane shear stresses.

The structure of the previous chapter is as follows: in Subchapter 3.1 is presented the representation of incremental elastic and electrical fields through two complex electric potentials. In Subchapters 3.2 and 3.3 are highlighted original products obtained. In Subchapter 3.2 using the boundary conditions and the method of conformal representation we determine the general expressions of complex potentials as Laurent series, and in Subchapter 3.3 we determine the coefficients of complex potentials in the case of an elliptical crack in a prestressed and prepolarized piezoelectric material of $\bar{4}2m$ class. The novelty of this chapter is the representation of an elementary solution in the form of a compact form of the complex potentials and using this, we determine the incremental sterss and electric fields in a prestressed and prepolarized piezoelectric material acted on large distances by constant and uniform antiplane shear stresses.

The results of this chapter were published in the paper [21] Craciun, EM., Ghita, GMD., Rapeanu, E. Prestressed and prepolarized piezoelectric material with an elliptical hole. Z. Angew. Math. Phys. 76, 18 (2025).

Chapter 4

Bridge crack propagation in a prestressed and prepolarized piezoelectric material

The main goal of this chapter consists in presenting new results regarding the mathematical modeling of a bridge crack in prestressed and prepolarized piezoelectric materials in mode III of classical fracture.

The structure of this chapter is as follows: In Subchapter 4.1, the antiplane state of prestressed and prepolarized piezoelectric materials is presented, studying the case of PZT type prestressed and prepolarized piezoelectric materials. In Subchapters 4.1, 4.2 and 4.3 the original results are obtained. In Subchapter 4.2 we study the problem of an antiplane bridge crack in prestressed and prepolarized piezoelectric materials. Using the boundary conditions of the bridgeded crack, we get linear differential nonhomogeneous equations having the complex potentials as unknown. For a constant value of the applied incremental loads, we will determine the complex potentials, displacements and incremental stress fields corresponding to the third mode of classical fracture. In Subchapter 4.3 we will extend Sih's strain energy density (SED) fracture criterion in the case of prestressed and prepolarized piezoelectric materials and we will study the propagation of a bridge crack in antiplane mode of fracture in a piezoelectric material of PZT type. From numerical results and the graphic representation of the density of the incremental deformation energy, we find the crack propagation direction as a function of different values of the stiffness constant as well as of different values of the initial applied elastic and electric fields.

The results of this chapter were published in the paper [25] Ghita, GMD., Craciun, E.M.: Reinforced crack propagation in a prestressed and prepolarized piezoelectric material, Compos. Struct. 2024;342:118248.

Chapter 5

Mathematical modeling of dynamic piezothermoelastic nanobars

The main goal of this chapter is to obtain new results regarding the mathematical modeling of one-dimensional piezoelectric materials in the context of the theory of generalized piezothermoelasticity.

The structure of this chapter is as follows: Subchapter 5.1 presents the basic equations, the constitutive relations and the heat conduction equation for an anisotropic piezo-thermoelastic medium. In Subchapters 5.1 and 5.2 the original results obtained are highlighted. In Subchapter 5.2, we determine the solution to our problem in the case of a homogeneous, transversely isotropic piezothermoelastic nanobar with a rectangular section subjected to the action of a uniformly distributed harmonic type load in a state of rest with fixed ends and subjected to a constant temperature. For different particular cases in the case of lead zirconate titanate piezoelectric material (PZT-5A), the influence of the frequency effect can be observed in the representation of the solutions obtained for the lateral deviation, potential thermal moment, electrical damping. thermoelasticity and of the frequency shift versus the length of the nanobar within the framework of the coupled theory (CT), the Lord-Shulman theory (LS) and the respective Green-Lindsey (GL) theory.

The results of this chapter were published in the paper [26] Kaur, I., Singh, K., Ghita, GMD.: New analytical method for dynamic response of thermoelastic damping in simply supported generalized piezothermoelastic nanobeam, ZAMM-Z. Angew. Math. Me. 2021;101(10):e202100108.

Chapter 6

Conclusions

Structure of the doctoral thesis

This doctoral thesis, entitled ***Mathematical modeling of crack propagation in piezoelectric materials with initial fields***, is structured in six chapters, followed by a bibliographical list containing 84 titles .

Original results

The original contributions, included in this doctoral thesis, are the following:

1. The study included in Chapter 3, entitled *Prestressed and prepolarized piezoelectric material with an elliptical crack* was published in the paper [21].

2. The study included in Chapter 4, entitled *Bridge crack propagation in a prestressed and prepolarized piezoelectric material* was published in the paper [25].

3. The study included in Chapter 5, entitled *Mathematical modeling of dynamic piezothermoelastic nanobars* was published in the paper [26] and was extended for magnetopiezothermoelastic nanobars in the paper [27].

Disseminated results

Published papers during the doctoral studies

During the doctoral studies I published the following papers:

1. Craciun, EM., **Ghita, GMD.**, Rapeanu, E. Prestressed and prepolarized piezoelectric material with an elliptical hole. Z. Angew. Math. Phys. 2025;7618:18 , https://doi.org/10.1007/s00033-024-02396-4, ISI indexed journal, quartile Q2 (IF, AIS), see [21].

2. **Ghita, GMD.**, Craciun, E.M.: Reinforced crack propagation in a prestressed and prepolarized piezoelectric material, Compos. Struct. 2024;342:118248, https://doi.org/10.1016/j.compstruct.2024.118248, ISI indexed journal, quartile Q1 (IF, AIS), see [25].

3. Kaur, I., Singh, K., **Ghita, GMD.**: New analytical method for dynamic response of thermoelastic damping in simply supported generalized piezothermoelastic nanobeam, ZAMM-Z Angew Math Me. 2021;101(10):e202100108, https://doi.org/10.1002/zamm.202100108, ISI indexed journal, quartile Q1 (IF), see [26].

4. Kaur, I., Singh, K., **Ghita, GMD.**, Craciun, EM.: Modeling of a magneto-electro-piezo-thermoelastic nanobeam with two temperature subjected to ramp type heating, Proc. Rom. Acad., Ser. A: Math. Phys. Tech. Sci. Inf. Sci 2022;23(2):143-152, ISI indexed journal, quartile Q3 (IF, AIS), see [27].

Presentation of scientific research results

International and national conferences where I presented the scientific research results during doctoral studies, are the following:

- **Students' International Conference, CERC, Bucureşti, 06-07 Noiembrie, 2020**, Academia Tehnică Militară Ferdinand I București, with the paper *Cracks in a prestresed and prepolarized piezoelectric material.*
- **Conferința națională a studenților, masteranzilor și doctoranzilor Tehnonav Jr., Ediția a-XI-a, 25 Mai, 2022**, Universitatea Ovidius din Constanța, with the paper *Modelarea matematica a nanobarelor magneto-electro-piezotermoelastice.*
- **15-th International Conference on advanced computational engineering and Experimenting, ACEX 2022, Florența, Italia, 03-07 Iulie, 2022**, with the paper nr. ACEX 420 *Anti-plane interface crack in piezoceramics with initial fields*, section *Plasticity and Constitutive Modelling (SS2)*, online, 06 July 2022, hour 10:20-10:50.

- **27th International Conference on Composite Structures, ICCS27, Ravenna, Italia, 03-06 Septembrie, 2023**, University of Bologna, with the paper nr. 1224 *Reinforced crack propagation in a prestressed and prepolarizedpiezoelectric material*, section *Delamination, damage, fracture, failure and durability of composites* online, 06 September 2024 hour 11:50-12:10.

Awarding of scientific research results

The following article was awarded, within the national competition "Awarding of Research Results - UEFISCDI":

1. **New analytical method for dynamic response of thermoelastic damping in simply supported generalized piezothermoelastic nanobeam, ZAMM-Z. Angew. Math. Me. 2021;101(10):e202100108**, see [26].

If the "Awarding of Research Results - UEFISCDI" competition continues, the following two articles listed in Subchapter 6.3.1 will be awarded, being in the quartiles Q2 (AIS yellow zone) and Q1 (AIS red zone), respectively:

1. Craciun, EM., **Ghita, GMD.**, Rapeanu, E. Prestressed and prepolarized piezoelectric material with an elliptical hole. Z. Angew. Math. Phys. 2025;7618:18 , https://doi.org/10.1007/s00033-024-02396-4, ISI indexed journal, quartile Q2 (IF, AIS), see [21].

2. **Ghita, GMD.**, Craciun, E.M.: Reinforced crack propagation in a prestressed and prepolarized piezoelectric material, Compos. Struct. 2024;342:118248, https://doi.org/10.1016/j.compstruct.2024.118248, ISI indexed journal, quartile Q1 (IF, AIS), see [25].

Citări

The articles published during the doctoral studies, listed in Subchapter 6.3.1, had 29 citations in the Web of Science, the number of citations for each paper being mentioned below:

1. Craciun, EM., **Ghita, GMD.**, Rapeanu, E. Prestressed and prepolarized piezoelectric material with an elliptical hole. Z. Angew. Math. Phys. 2025;7618:18 , https://doi.org/10.1007/s00033-024-02396-4,ISI indexed journal, quartile Q2, see [21],**1 quoting**.

2. **Ghita, GMD.**, Craciun, E.M.: Reinforced crack propagation in a prestressed and prepolarized piezoelectric material, Compos. Struct. 2024;342:118248, https://doi.org/10.1016/j.compstruct.2024.118248, ISI indexed journal, quartile Q1, see [25], **2 quotings**.

3. Kaur, I., Singh, K., **Ghita, GMD.**: New analytical method for dynamic response of thermoelastic damping in simply supported generalized piezothermoelastic nanobeam, ZAMM-Z. Angew. Math. Me. 2021;101(10):e202100108, https://doi.org/10.1002/zamm.202100108, ISI indexed journal, quartile Q2, see [26], **19 quotings**.

4. Kaur, I., Singh, K., **Ghita, GMD.**, Craciun, EM.: Modeling of a magneto-electro-piezo-thermoelastic nanobeam with two temperature subjected to ramp type heating, Proc. Rom. Acad., Ser. A: Math. Phys. Tech. Sci. Inf. Sci 2022;23(2):143-152, ISI indexed journal, quartile Q3, see [27], **7 quotings**.

Future research directions

1. In the future, we intend to study prestressed and prepolarized piezoelectric materials with elliptical holes/cracks, using the conformal mapping method and theory of complex potentials in the case of prestressed thermopiezoelectric materials and for anisotropic magnetoelectroelastic materials. We also intend to study the interaction between an elliptical hole and a classical crack in the above mentioned materials, as well as to extend the study from an elliptical hole/crack with stress-free faces to the case of a bridged elliptical hole/crack at the interface between two prepolarized piezoelectric materials.

2. We will extend the current studies to the cases of interaction of bridged collinear or/and parallel bridged cracks, in the case of prestressed anisotropic composite materials, (see [15]-[20]) for prestressed and prepolarized piezoelectric materials and to prestressed and prepolarized magnetothermopiezoelectric materials.

Bibliography

[1] Abd-Elaziz E.M., Othman, M.I.A.: Effect of Thomson and thermal loading due to laser pulse in a magneto-thermo-elastic porous medium with energy dissipation. ZAMM-Z. Angew. Math. Me. 2019;99(8):e201900079.

[2] Abouelregal, A.E., Zenkour, A.M.: Thermoelastic response of nanobeam resonators subjected to exponential decaying time varying load. J. Theor. Appl. Mech. 2017;55(3):937-948.

[3] Ansari, R., Gholami, R.: Nonlocal nonlinear first-order shear deformable beam model for postbuckling analysis of magneto-electro-thermo elastic nanobeams. Sci. Iran.2016;23(6):3099-3114.

[4] Baesu, E., Soos, E.: Antiplane piezoelectricity in the presence of initial mechanical and electrical fields, Math. Mech. Solids 2001;6: 409-422.

[5] Baesu, E., Soos, E.: Antiplane fracture in a prestressed and prepolarized piezoelectric crystal, IMA J. of Appl. Math. 2001;66:449-508.

[6] Bardzokas, D., Filshtinsky, M.L.: Concentration of electroelastic fields in a composite piezoceramic plate with a hole intersecting the interface of materials. Mech. Compos. Mater. 1999;35:249–252.

[7] Bardzokas, D., Rodriguez, R., Filishtinskii, M.L.: Extension of a piezoceramic bimorph with a crack crossing the interface. Mech. Compos. Mater. 1997;33:338–342.

[8] Bardzokas, D., Filshtinsky, M.L., Rodriguez, R.: Concentration of electroelastic fields in a composite piezoceramic plate with defects crossing the interface. Mech. Compos. Mater. 1998;34:549–556.

[9] Bardzokas, D., Filshtinsky, M.L., Rodriguez, R.:Concentration of electroelastic fields in a composite piezoceramic plate with a hole intersecting the interface of materials. Mech. Compos. Mater. 1999;35:249–252.

[10] Bertoldi, K., Bigoni, D., Drugan, W.J.: A discrete-fibers model for bridged cracks and reinforced elliptical voids, J. Mech. Phys. Solids 2007;55:1016-1035.

[11] Bigoni, D.: Nonlinear Solid Mechanics, Bifurcation Theory and Material Instability, Cambridge University Press, New York, 2012.

[12] Bigoni, D., Movchan, A.B.: Statics and dynamics of structural interfaces in elasticity. Int. J. Solids Struct. 2002;39:4843-4865.

[13] Chen, Q., Wang, G. Computationally-efficient homogenization and localization of unidirectional piezoelectric composites with partially cracked interface. Compos. Struct. 2020;232:111452.

[14] Cristescu, N.D., Craciun, E.M., Soos, E.: Mechanics of Elastic Composites, CRC Press, Boca Raton, FL, 2004.

[15] Craciun, E.M., Soos, E.: Interaction of two unequal cracks in a pre-stressed fiber reinforced composite. Int. J. Fract. 1998;94:137–159.

[16] Craciun, E.M., Sadowski, T., Rabaea, A.: Stress concentration in an anisotropic body with three equal collinear cracks in Mode II of fracture. ZAMM · Z. Angew. Math. Mech. 2014;94:721–729.

[17] Craciun, E.M., Barbu, L.: Compact closed form solution of the incremental plane states in a prestressed elastic composite with an elliptical hole, Z Angew. Math. Mech., ZAMM 2013;95(2):193-199.

[18] Craciun, E.M., Soos, E.: Antiplane states in an elastic body containing an elliptical hole. Crack propagation, Math. Mech. Solids 2006;11: 459-466.

[19] Craciun, E.M., Baesu, E., Soos, E.: General solution in terms of complex potentials in antiplane states in prestressed and prepolarized piezoelectric crystals: application to Mode III fracture propagation, IMA J of Appl. Math. 2005;70:39-52.

[20] Craciun, E.M., Rabaea, A., Das, A.: Cracks interaction in a pre-stressed and pre-polarized piezoelectric material. J. Mech. 2020;36:177–182.

[21] Craciun, E.M., Ghita, G.M.D., Rapeanu, E.: Prestressed and prepolarized piezoelectric material with an elliptical hole, Z. Angew. Math. Phys. 2025;76:18.

[22] Ebrahimi, F., Barati, M.R.: Dynamic modeling of a thermo-piezo-electrically actuated nanosize beam subjected to a magnetic field. Appl. Phys. A 2016;122(4):451.

[23] Eom, C.B., Trolier-McKinstry, S.: Thin-film piezoelectric MEMS. MRS Bull.2012;37(11):1007-1017.

[24] Eringen, A.C., Maugin, G.A.: Electrodynamics of Continua, Vol.1, Foundations and Solid Media, Springer, New York, 1990.

[25] Ghita, G.M.D., Craciun, E.M.: Reinforced crack propagation in a prestressed and prepolarized piezoelectric material, Compos. Struct. 2024;342:118248.

[26] Kaur, I., Singh, K., Ghita, G.M.D.: New analytical method for dynamic response of thermoelastic damping in simply supported generalized piezothermoelastic nanobeam, ZAMM-Z. Angew. Math. Me. 2021;101(10):e202100108.

[27] Kaur, I., Singh, K., Ghita, G.M.D., Craciun, EM.: Modeling of a magneto-electro-piezo-thermoelastic nanobeam with two temperature subjected to ramp type heating, Proc. Rom. Acad., Ser. A: Math. Phys. Tech. Sci. Inf. Sci 2022;23(2):143-152.

[28] Guz, A.N.: Fundamentals of the Three Dimensional Theory of Stability of Deformable Bodies, Springer-Verlag, Berlin, Heidelberg, 1999.

[29] Gherrous, M., Ferdjani, H.: Analysis of a Griffith crack at the interface of two piezoelectric materials under anti-plane loading, Continuum Mech. Thermodyn. 2016;28:1683–1704.

[30] Huang, Z.Y., Bao, R.H., Bian Z.G.: The potential theory method for a half-plane crack and contact problems of piezoelectric materials. Compos. Struct. 2007;78(4):596-601.

[31] Kaur, I., Lata, P.: Rayleigh wave propagation in transversely isotropic magneto-thermoelastic medium with three-phase-lag heat transfer and diffusion. Int. J. Mech. Mater. Eng. 2019;14:12.

[32] Kaur, I., Lata, P.: Stoneley wave propagation in transversely isotropic thermoelastic medium with two temperature and rotation. GEM - Int. J. Geomathematics 2020;11(1): 1-17.

[33] Kaur, I., Lata, P.: Effect of Hall current in transversely isotropic magneto-thermoelastic rotating medium with fractional-order generalized heat transfer due to ramp-type heat. Indian J. Phys. 2021;95:1165-1174.

[34] Kaur, I., Lata, P.: Effect of Hall current on propagation of plane wave in transversely isotropic thermoelastic medium with two temperature and fractional order heat transfer. SN Appl. Sci. 2019;1(8):900.

[35] Kaur, I., Lata, P.: Transversely isotropic thermoelastic thin circular plate with constant and periodically varying load and heat source. Int. J. Mech. Mater. Eng. 2019;14(1):10.

[36] Khisaeva Z.F., Ostoja-Starzewski, M.: Thermoelastic Damping in Nanomechanical Resonators with Finite Wave Speeds. J. Therm. Stress. 2006;29(3): 201-216.

[37] Kozinov, S., Sheveleva, A., Loboda, V.: Fracture behavior of periodically bonded interface of piezoelectric bi-material under compressive–shear loading. Math. Mech. Solids. 2019;24(10):3216-3230.

[38] Kumar R., Sharma, P.: Modelling of piezothermoelastic beam with fractional order derivative. Curved Layer. Struct. 2016;3(1):e0009.

[39] Iesan, D.: Plane strain problems in piezoelectricity. Int. J. Eng. Sci.1997; 25(11-12):1511-1523.

[40] Lata, P., Kaur, I.: Thermomechanical interactions due to time harmonic sources in a transversely isotropic magneto thermoelastic solids with rotation. Int. J. Microstruct. Mater. Prop. 2019;14(6):549-57.

[41] Lata, P., Kaur, I.: A Study of Transversely Isotropic Thermoelastic Beam with Green-Naghdi Type-II and Type-III Theories of Thermoelasticity. Appl. Appl. Math. An Int. J. 2019;14(1):270-283.

[42] Lekhnitski, S.G.: Theory of Elasticity of Anisotropic Elastic Body, Holden Day, San Francisco, 1963.

[43] Li, D., He, T.: Investigation of generalized piezoelectric-thermoelastic problem with nonlocal effect and temperature-dependent properties. Heliyon. 2018;4(10):e00860.

[44] Liang X., Shen, S.: Effect of electrostatic force on a piezoelectric nanobeam. Smart Mater. Struct. 2012;21(1):015001.

[45] Lifshitz, R., Roukes, M.L.: Thermoelastic damping in micro- and nanomechanical systems. Phys. Rev. B. 2000;61(8):5600-5609.

[46] Loboda, V.V., Herrmann K.P.: A contact zone approach for an interface crack in a piezoelectric anisotropic bimaterial. ZAMM · Z. Angew. Math. Mech. 2000;80(S2):479-480.

[47] Ma, L., Wu, L.Z., Zhou, Z.G., Guo, L.C.: Scattering of the harmonic anti-plane shear waves by a crack in functionally graded piezoelectric materials. Compos. Struct. 2005;69(4):436-441.

[48] Malikan, M., Eremeyev, V.A.: Flexomagnetic response of buckled piezomagnetic composite nanoplates. Compos. Struct. 2021;267:113932.

[49] Mohamed Othman, I. A., Atwa, S. Y., Waheed, M. H., Ethar, A. A.: Propagation of Plane Waves in Generalized Piezo-thermoelastic Medium: Comparison Of Different Theories, Int. J. Innov. Res. Sci. Eng. Technol. 2015; 4(4), 2292–2300.

[50] Muskhelishvili, N.I.: Some Basic Problems of the Mathematical Theory of Elasticity, Noordhoff Ltd Groningen, 1953.

[51] Qin, Q.H., Mai, Y.W.: Crack path selection in piezoelectric bimaterial. Compos. Struct. 1999;47(1-4):519-524.

[52] Peride, N., Carabineanu, A., Craciun, E.M.: Mathematical modelling of the interface crack propagation in a prestressed fiber reinforced elastic composite. Comp. Mater. Sci. 2009;45:684-692.

[53] Pinchas, B., Avrashi, J.: Interlaminar stress analysis for laminated plates containing a curvilinear hole. Computers & Structures 1985;21:917–32.

[54] Pryce, L., Morini, L., Mishuris, G.: Weight function approach to a crack propagating along a bimaterial interface under arbitrary loading in an anisotropic solid. JOMMS. 2013;8(8):479-500.

[55] Rao, S.S.: Vibration of continuous systems. New Jersey: John Wiley & sons., 2007.

[56] Sadek, I., Abukhaled, M.: Optimal control of thermoelastic beam vibrations by piezoelectric actuation. J. Control Theory Appl. 2013;11(3):463-467.

[57] Sigaeva, T., Schiavone, P.: The effect of surface stress on an interface crack in linearly elastic materials. Math. Mech. Solids. 2016;21(6):649-656.

[58] Singh, R., Das, S.: Analysis of multiple parallel cracks in a functionally graded magneto-electro-elastic plane using boundary collocation method. Arch. Appl. Mech. 2023;93:4497-4516.

[59] Sih, G.C., Yu, H.Y.: Volume fraction effect of magnetoelectroelastic composite on enhancement and impediment of crack growth. Compos. Struct. 2005;68(1):1-11.

[60] Singh, B.M., Rokne, J.G., Dhaliwal, R.S.: Closed form solution for an annular elliptic crack around an elliptic rigid inclusion in an infinite solid, ZAMM 2012;92(11-12):882-887.

[61] Sharma J.N., Kaur, I.: Transverse vibrations in thermoelastic-diffusive thin micro-beam resonators. J. Therm. Stress. 2014;37(11):1265-1285.

[62] Sharma, J.N., Kaur, R.: Modeling and analysis of forced vibrations in transversely isotropic thermoelastic thin beams. Meccanica 2015;50(1):189-205.

[63] Shen, S., Nishioka, T.: Fracture of piezoelectric materials: energy density criterion. Theor. Appl. Fract. Mech. 2000;33:57-65.

[64] Soos, E.: Stability, resonance and stress concentration in prestressed piezoelectric crystals containing a crack, Int. J. Eng. Sci. 1996;34:1647-1673.

[65] Sosa, H., Khutoryansky, N.: New developments concerning piezoelectric material with defects. Int. J. Solids Struct. 1996;33(23):3399-3414.

[66] Sosa, H: Plane problems in piezoelectric media with defects. Int. J. Solids Struct. 1991;28(4):491-505.

[67] Sosa, H: On the fracture mechanics of piezoelectric solids, Int. J. Solids Struct. 1992;29(21):2613-2622.

[68] Sosa, H., Pak, Y.E.: Three-dimensional eigenfunction analysis of a crack in a piezoelectric material. Int. J. Solids Struct. 1990;26(1):1-15.

[69] Suo, Z., Kuo, C. M., Barnett, D. M., Willis J. R.: Fracture mechanics for piezoelectric ceramics. J. Mech. Phys. Solids. 1992;40:739–765.

[70] Tvardovski, V.V. : Further results on rectilinear line cracks and inclusions in anisotropic medium, Theor. Appl. Fracture Mech. 1990;13:193-207.

[71] Vahdat, A.S., Rezazadeh, G., Ahmadi, G.: Thermoelastic damping in a micro-beam resonator tunable with piezoelectric layers. Acta Mech. Solida Sin. 2012; 25(1):73-81.

[72] Valentini, M., Serkov, S.K., Bigoni, D., Movchan, A.B.: Crack propagation in a brittle elastic material with defects. J. Appl. Mech.1999;66:79-86.

[73] Zenkour, A.M.: Refined two-temperature multi-phase-lags theory for thermomechanical response of microbeams using the modified couple stress analysis. Acta Mech. 2018;229(9):3671-3692.

[74] Zhu, S., Yu, H., Guo, L.: Analysis of an interfacial crack between two nonhomogeneous piezoelectric materials using a new domain-independent interaction integral. Compos. Struct. 2024;331:117873.

[75] Zuo, J.Z., Sih, G.C.: Energy density theory formulation and interpretation of cracking behavior for piezoelectric ceramics. Theor. Appl. Fract. Mech. 2000;34:17-33.

[76] Xiao, J., Xu, Y., Zhang, F: A rigorous solution for the piezoelectric materials containing elliptic cavity or crack with surface effect. ZAMM · Z. Angew. Math. Mech. 2015;96(5):633-641.

[77] Wang, X., Schiavone, P.: Electroelastic field for a blunt crack in an anisotropic piezoelectric material. Continuum Mech. Thermodyn. 2021;33:2509–2514.

[78] Wang, X., Zhou, K.: A crack with surface effects in a piezoelectric material. Math. Mech. Solids. 2017;22(1):3-19.

[79] Yan, Z., Feng, W.J., Zhang C.: Interfacial crack growth in piezoelectric-piezomagnetic bi-layered structures with a modified mechanical energy release rate criterion. Compos. Struct. 2021;262:113344.

[80] Yang, W., Liu, M., Chen, S., Kang W., Chen J., Li, Y.: Electromechanical analysis of a self-sensing torsional micro-actuator based on CNTs reinforced piezoelectric composite with damage. Compos. Struct. 2023;313:116945.

[81] Yong, H.D., Zhou, Y.H.: A mode III crack in a functionally graded piezoelectric strip bonded to two dissimilar piezoelectric half-planes. Compos. Struct. 2007;79(3):404-410.

[82] Youssef, H. M., El-Bary A. A.: Theory of hyperbolic two-temperature generalized thermoelasticity. Mater. Phys. Mech. 2018;40(2):158-171.