“OVIDIUS” UNIVERSITY OF CONSTANŢA
DOCTORAL SCHOOL OF MATHEMATICS
DOMAIN - MATHEMATICS

Summary

Ph.D THESIS

Ph.D Coordinator
Prof. univ. dr. Eduard - Marius Crăciun

Ph.D Student
Anamaria - Nadie Emin (married Leonte)

CONSTANȚA, 2023

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NEW APPROACHES FOR THERMOELASTIC MEDIA WITH DOUBLE POROSITY

Ph.D Coordinator
Prof. univ. dr. Eduard - Marius Crăciun

Ph.D Student
Anamaria - Nadie Emin (married Leonte)

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Contents

Keywords

double porosity, thermoelasticity, uniqueness, reciprocity, Betti, Green-Lindsay, microtemperatures, existence

Chapter 1

Introduction

1.1 Short history

A porous material is a material that contains free, regular or irregular spaces in its structure. These free spaces appear in the mass of the material as pores, cracks or voids. The porosity of a material is given by the totality of these free spaces. The elasticity's of porous media theory also takes into consideration the volume fraction, which represents an additional degree of freedom for each microelement, thus, the elasticity's of porous media theory represents an extension of the classical theory of elasticity. Thermoelasticity deals with the study of the states of tension and deformations produced under the action of a thermal field. Uniqueness and various others aspects of the solutions of the problems considered in the context of elasticity and thermoelasticity have been intensively studied in the last decades.

The concept of double porosity was introduced for the first time by Barenblatt et al., [4] and by Barenblatt, [3]. Materials with double porosity were approached using various theories. Considering the Cowin-Nunziato theory, [11], for the materials with voids, Ieşan and Quintanilla, [20], developed a nonlinear theory for thermoelastic bodies with double porosity, considering the body pores, at the macro level and the cracks of the porous matrix, at the micro level. Svanadze, [34], obtained several uniqueness theorems for isotropic materials with double porosity. The boundary values problems in the context of the thermoelasticity of dipolar bodies were studied by Marin and Crăciun, [25], and Marin et al., [23]. Florea, [14], published some studies regarding the backward in time problems of this type of materials. Florea and Bobe, [15], approached the Moore-Gibson-Thompson theory in the context of bodies with double porosity.

Many researchers have shown their interest in the theory of Green-Lindsay thermoelasticity, due to the fact that this theory also takes into account the temperature's rate, as a constitutive variable, and allows the propagation of waves at finite speeds. This type of theory was approached from the perspective of classical thermoelasticity by Green and Lindsay, [16], from the perspective of thermoelastic solid by Nieto et al., [30], from the perspective of thermoviscoelasticity by Aouadi et al., [2], and from the perspective of dipolar bodies thermoelasticity by Marin et al., [26].

Ieşan and Quintanilla, [19], developed the linear theory of thermoelasticity with microtemperatures for materials with internal structure. In this theory, material particles, in addition to classical displacement and temperature fields, possess microtemperatures. Marin et al., [24], obtained results related to the existence and uniqueness of the solution in the theory of micropolar thermoelastic bodies whose microparticles possess microtemperatures, by transforming the mixed initial and boundary value problem in an equation of evolution on a Hilbert space. Bazzara et al., [5], used the operators semigroup theory for different media of continuum mechanics.

Svanadze, [33], studied the concept of triple porosity, where pores at macro, meso and micro level are considered.

The applications of materials with double porosity are spread over a wide range of fields, such as: civil and geotechnical engineering, biomechanics, seismology, geomagnetism and geodesy, energy production and petroleum engineering.

1.2 Purpose and objectives of the research

In this doctoral thesis, I propose to research and obtain new results related to:

1. The uniqueness of the solution of the mixed problem for thermoelastic materials with double porosity;
2. The uniqueness of the solution of the mixed problem for thermoelastic materials with double porosity, in the context of Green-Lindsay thermoelasticity;
3. The uniqueness of the solution of the mixed problem for thermoelastic materials with double porosity, when the influence of microtemperature is considered.

Each of the three studies listed above will represent a chapter of the doctoral thesis, containing original results. Along with the three chapters mentioned above, this doctoral thesis will include an introductory chapter, a chapter containing the preliminaries of the study, a chapter representing the conclusions of the study and a bibliographical list.

1.3 Acknowledgements

I would like to thank my Ph.D coordinator, Prof. univ. dr. Eduard - Marius Crăciun, who offered me guidance, encouragement and support throughout the years of my doctoral studies.

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Chapter 2

Preliminaries

The original results of this doctoral thesis are included in Chapters 3, 4 and 5, as follows: Chapter 3 contains results of uniqueness of the solution of the mixed problem for thermoelastic materials with double porosity; Chapter 4 contains results of uniqueness of the solution of the mixed problem for thermoelastic materials with double porosity, in the context of Green-Lindsay thermoelasticity; Chapter 5 contains results of uniqueness of the solution of the mixed problem for thermoelastic materials with double porosity, when the influence of microtemperature is taken into account.

In Chapters 3 and 4, I used Betti's Theorem, [6], [7].

In Chapter 4, I used the Laplace transform, the property of the original derivation and the image of the convolution product, [32], [29].

In Chapter 5, I used the contractions semigroup theory, the Hille-Yosida Theorem, the notion of dissipative operator and the Lumer-Phillips Corollary, [31].

Thus, in this chapter, I defined, stated and presented the results mentioned above, belonging to other authors, which I needed and which I used in obtaining my own results.

Chapter 3

Some uniqueness results for thermoelastic materials with double porosity

The main purpose of this chapter is to obtain new results regarding the uniqueness of the solution of the mixed problem with initial and boundary values for anisotropic thermoelastic materials with double porosity.

Thus, in Chapter 3, I included some new results related to the uniqueness of the solution of the mixed problem for thermoelastic materials with double porosity. This chapter contains two subchapters, as follows:

In Subchapter 3.1, I presented the basic equations that model the thermoelastic materials with double porosity, respectively the equations of motion, the balance of equilibrated forces, the energy equation and the constitutive equations. From the basic equations, I obtained the mixed problem corresponding to this type of structure, consisting of a system of differential equations, along with the initial and boundary conditions and I established the hypotheses necessary to demonstrate the uniqueness of the solution of the considered mixed problem.

In Subchapter 3.2, I presented the main results of the chapter. In the first part of the subchapter, I introduced two functions for which, in Theorem 3.2.1, I demonstrated an identity corresponding to the difference between them. In Theorem 3.2.2, I found two other expressions for the two functions introduced. Based on Theorems 3.2.1 and 3.2.2, I obtained a first result regarding the uniqueness of the solution of the mixed problem presented in Subchapter 3.1. More precisely, in Theorem 3.2.3, I demonstrated that the mixed problem consisting of the system of differential equations, satisfying the initial and boundary conditions and the hypotheses considered in Subchapter 3.1, admits at most one solution. In order to obtain a new result regarding the uniqueness of the solution of the mixed problem presented in Subchapter 3.1, I introduced two new functions, continuous in time, I defined their convolution product, I considered two systems of external loads with their corresponding solutions and, in Lemma 3.2.4, I obtained a commutativity relation, using the defined convolution product. Starting from Lemma 3.2.4, in Theorem 3.2.5, I demonstrated a Betti-type reciprocity relation, [6], [7], which establishes a correspondence between the two considered systems of external loads and the solutions of the respective loads. Furthermore, I demonstrated a differential relation whose expression I included in Lemma 3.2.6. Finally, using Lemma 3.2.6 and Theorem 3.2.5, in Theorem 3.2.7, I obtained a new result regarding the uniqueness of the solution of the mixed problem consisting of the system of differential equations, satisfying the initial and boundary conditions and the hypotheses considered in Subchapter 3.1, including in addition, the condition that the thermal conductivity tensor is positive defined.

The results of this chapter were published in A.N. Emin, O.A. Florea, E.M. Crăciun, [13].

Chapter 4

Green-Lindsay thermoelasticity for materials with double porosity

The main purpose of this chapter is to obtain new results regarding the uniqueness of the solution of the mixed problem with initial and boundary values for thermoelastic materials with double porosity, starting from the classical theory of Green-Lindsay elasticity, [16].

Thus, in Chapter 4, I included some new results related to uniqueness of the solution of the mixed problem for thermoelastic materials with double porosity, in the context of Green-Lindsay thermoelasticity, [16]. This chapter contains two subchapters, as follows:

In Subchapter 4.1, I presented the basic equations that model the thermoelastic materials with double porosity using Green-Lindsay thermoelasticity, [16], respectively the components of the heat flow, the equations of motion, the balance of equilibrated forces, the energy equation and the constitutive equations. From the constitutive equations and the components of the heat flow, I obtained the quadratic form of the generalized free energy. I presented the mixed problem corresponding to this type of structure consisting of the equations of motion, the balance of equilibrated forces, the energy equation and the constitutive equations, along with the initial and boundary conditions.

In Subchapter 4.2, I presented the main results of the chapter. In the first part of the subchapter, I considered two systems of external loads which act on the thermoelastic body with double porosity and the thermoelastic states corresponding to them. Furthermore, I introduced two functions and I defined their convolution product. Applying the Laplace transform, [32], [29], using the property of the original derivation, [32], [29] and using the defined convolution product, in Theorem 4.2.1, I demonstrated a Betti-type reciprocity relation, [6], [7], between the two systems of external loads. Afterwards, I used the energy function proposed by Biot, [8] and the scalar function expressed by Green and Lindsay, [16], thus obtaining the quadratic form of the energy function according to Biot. Taking into consideration the result of Theorem 4.2.1 and the quadratic form of the energy function according to Biot, in Theorem 4.2.2, I found the energy form in the Green-Lindsay context, [16], for materials with double porosity. Finally, in Theorem 4.2.3, using Theorem 4.2.2, I demonstrated that the mixed problem presented in Subchapter 4.1, admits at most one solution, if the quadratic form of the energy function according to Biot is positive defined.

The results of this chapter were published in A.N. Emin, O.A. Florea, [12].

Chapter 5

Microtemperature's effects on thermoelastic materials with double porosity

The main purpose of this chapter is to obtain new results regarding the uniqueness, existence and continuous dependence of the solution of the mixed problem with initial and boundary values for thermoelastic materials with double porosity, when the influence of microtemperature is taken into account.

Thus, in Chapter 5, I included some new results regarding the uniqueness, the existence and continuous dependence of the solution of the mixed problem for thermoelastic materials with double porosity, when the influence of microtemperature is taken into account. This chapter contains two subchapters, as follows:

In Subchapter 5.1, I presented the basic equations that model the anisotropic thermoelastic materials with double porosity and microtemperatures, respectively the thermal displacement introduced by Green and Naghdi, [17], the microthermal displacement, the equations of motion, the balance of equilibrated forces and the energy equations taking into account the microtemperatures. I defined the quadratic form of the internal energy in the context of linear thermoelasticity, which I considered positive defined. From the quadratic form of the internal energy, based on the Ieşan method, [21], I obtained the constitutive equations. From the constitutive equations, the equations of motion, the balance of equilibrated force and the energy equations, I obtained the mixed problem for thermoelastic materials with double porosity and microtemperatures, consisting of a system of differential equations, along with the initial and boundary conditions and I established the hypotheses necessary to demonstrate the desired results.

In Subchapter 5.2, I presented the main results of the chapter. In order to obtain a result regarding the uniqueness of the solution of the mixed problem presented in Subchapter 5.1, I deduced an auxiliary result. Thus, in Lemma 5.2.1, I obtained the expression of the deformation of a thermoelastic material with double porosity and microtemperatures. With help of Lemma 5.2.1 and taking into account the hypotheses imposed in Subchapter 5.1, I obtained, in Theorem 5.2.2, a result regarding the uniqueness of the solution of the mixed problem with initial and boundary values when the microtemperature influence is considered. In order to study the existence of the solution, due to the complexity of the system of differential equations which, along with the initial and boundary conditions, constitute the mixed problem, I transformed the mixed problem into a Cauchy-type problem. In this situation, I limited the study to the case when the boundary conditions are null and I defined a Hilbert space, [9], [31], [27], [28], [1], on which I considered an equation of evolution and two vector functions and I defined their scalar product. Based on Korn's first inequality, [10], [18], [35], in Theorem 5.2.3, I demonstrated that the norm of the scalar product of the two vector functions introduced is equivalent to the norm of the defined Hilbert space. From the system of differentials equations obtained in Subchapter 5.1, using the procedure presented by Ieşan, [21], I extracted the operators based on which I introduced a matrix operator. In Theorem 5.2.4, I proved that the introduced matrix operator is dissipative. In Theorem 5.2.5, using the introduced matrix operator and applying the Lax-Milgram Theorem, [9], [36], [31], I ensured the existence of the solution of the Cauchy problem. Through Theorems 5.2.4 and 5.2.5 I showed that the introduced matrix operator fulfills the conditions of the Lumer-Phillips Corollary, [31], corresponding to the Hille-Yosida Theorem, [31], so that, in Theorem 5.2.6 I was able to demonstrate that the introduced operator generates a semigroup of contractions, [31], on the previously defined Hilbert space. Finally, based on Theorem 5.2.6, I proved, in Theorem 5.2.7, that the solution of the considered Cauchy problem is unique, and In Theorem 5.2.8, using the Lumer-Phillips Corollary, [31], I demonstrated the continuous dependence of the considered Cauchy problem solution. In Theorems 5.2.5 - 5.2.8 I took into consideration the hypotheses considered in Subchapter 5.1.

Chapter 6

Conclusions

6.1 Structure of the doctoral thesis

Considering the purpose and objectives of the research (see Subchapter 1.2), I structured the present doctoral thesis, entitled *New approaches for thermoelastic media with double porosity*, in six chapters, followed by a bibliographical list containing 111 titles.

6.2 Original results

1. The study included in Chapter 3, entitled *Some uniqueness results for thermoelastic materials with double porosity*, published in [13],
2. The study included in Chapter 4, entitled *Green-Lindsay thermoelasticity for materials with double porosity*, published in [12],
3. The study included in Chapter 5, entitled *Microtemperature's effects on thermoelastic materials with double porosity*.

6.3 Disseminated results

6.3.1 Articles published during the doctoral studies, included in the doctoral thesis

1. **A. N. Emin**, O. A. Florea, E. M. Crăciun, Some Uniqueness Results for Thermoelastic Materials with Double Porosity Structure, Continuum Mechanics and Thermodynamics, 33(4), 1083-1106, (2021), https://doi.org/10.1007/s00161-020-00952-7, ISI indexed journal, see [13],

2. **A. N. Emin**, O. A. Florea, Green-Lindsay Thermoelasticity for Double Porous Materials, The Journal of "Ovidius" University of Constanta, Mathematics Series, 31(1), 97-113, (2023), https://doi.org/10.2478/auom-2023-0005, ISI indexed journal, see [12].

6.3.2 Presentation of scientific research results

- **Students' International Conference CERC, Bucureşti, 23-25 April 2019**
  Organizer: **Military Technical Academy "Ferdinand I", Bucharest**

- **International Conference on Nonlinear Solid Mechanics, Roma, 16-19 June 2019**
  Organizer: **International Research Center on Mathematics and Mechanics of Complex Systems of the University of L'Aquila**

- **Students' International Conference CERC, Bucureşti, 06-07 November 2020**
  Organizer: **Military Technical Academy "Ferdinand I", Bucharest**

- **4th International Conference of the Doctoral School TUIASI, Iaşi, 19-21 May 2021**
  Organizer: **"Gheorghe Asachi" Technical University of Iaşi**

- **12th International Conference TIMA21 - Innovative Technologies for Joining Advanced Materials, Timişoara, 25-26 November 2021**
  Organizer: **National Institute of Research - Development in Welding and Testing of Materials ISIM Timişoara**

- **Mathematical Communication Session, Constanţa, 11 December 2021**
  Organizer: **Faculty of Mathematics and Informatics, "Ovidius" University of Constanţa**

6.3.3 Awarding of scientific research results

The following article was awarded, within the national competition "Research results award - UEFISCDI":

- **Some Uniqueness Results for Thermoelastic Materials with Double Porosity Structure, Continuum Mechanics and Thermodynamics, PN-III-P1-1.1-PRECISI-2021-61237,** see [13]

6.4 Future research directions

1. New results related to the uniqueness of the solution of the mixed problem with initial and boundary values for materials with triple porosity.

2. The influence of the use of materials with double and triple porosity in the design of variable geometry floating bodies for catamarans.

Bibliographical references

[1] Adams, R.A., *Sobolev Spaces*, Int. J. Pure. Appl. Math., 1st ed., Academic Press: New York, NY, USA, 65, (1975)

[2] Aouadi, M., Campo, M., Copetti, M.I.M., Fernández, J.R., *Analysis of a multidimensional thermoviscoelastic contact problem under the Green–Lindsay theory*, J. Comput. Appl. Math., 345, 224–246, (2019)

[3] Barenblatt, G.I., *On certain boundary-value-problems for the equations of seepage of liquid in fissured rocks*, Prikl. Mat. Mekh., 27(2), 513–518, (1963)

[4] Barenblatt, G.I., Zheltov, Iu.P., Kochina, I.N., *Basic concept in the theory of seepage of homogeneous liquids in fissured rocks (strata)*, J. Appl. Math. Mech., 24(5), 1286-1303, (1960)

[5] Bazarra, N., Fernández, J.R., Quintanilla, R., *Lord - Shulman thermoelasticity with microtemperatures*, Appl. Math. Optim., 84, 1667-1685, (2021)

[6] Bănuţ, V., Teodorescu, M.E., *Statica construcţiilor. Aplicaţii. Structuri static nedeterminate*, Matrix Rom, (2003)

[7] Betti, E., *Teoria dell'elasticità*, Il nuovo cimento, 7, 69-97, (2003)

[8] Biot, M.A., *General theory of three-dimensional consolidation*, J. Appl. Phys., 12, 155-164, (1941)

[9] Brezis, H., *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer: New York, NY, USA, (2011)

[10] Ciarlet, P.G., *On Korn's inequality*, Chin. Ann. Math. Ser. B, 31, 607–618, (2010)

[11] Cowin, S.C., Nunziato, J.W., *Linear elastic materials with voids*, J. Elast., 13, 125-147, (1983)

[12] Emin, A.N., Florea, O.A., *Green-Lindsay thermoelasticity for double porous materials*, An. Şt. Univ. Ovidius Constanţa, 31(1), 97-113, (2023)

[13] Emin, A.N., Florea, O.A., Crăciun, E.M., *Some uniqueness results for thermoelastic materials with double porosity structure*, Continuum Mech. Thermodyn., 33, 1083–1106, (2020)

[14] Florea, O., *The backward in time problem of double porosity material with microtemperature*, Symmetry, 11(4), 552, (2019)

[15] Florea, O.A., Bobe, A., *Moore – Gibson – Thompson thermoelasticity in the context of double porous materials*, Continuum Mech. Thermodyn., 33, 2243–2252, (2021)

[16] Green, A.E., Lindsay, K.A., *Thermoelasticity*, J. Elast., 2, 1-7, (1972)

[17] Green, A.E., Naghdi, P.M., *Thermoelasticity without energy dissipation*, J. Elast., 31(3), 189-208, (1993)

[18] Ieşan, D., *A theory of thermoelastic materials with voids*, Acta Mech., 60, 67-89, (1986)

[19] Ieşan, D., Quintanilla, R., *On a theory of thermoelasticity with microtemperatures*, J. Therm. Stress., 23(3), 199-215, (2000)

[20] Ieşan, D., Quintanilla, R., *On a theory of thermoelastic materials with a double porosity structure*, J. Therm. Stress., 37(9), 1017-1036, (2014)

[21] Ieşan, D., Quintanilla, R., *On thermoelastuc bodies with inner structure and microtemperatures*, J. Math. Anal. Appl., 351(1), 12-23, (2009)

[22] Lakes, R.S., *Physical meaning of elastic constants in Cosserat, void, and microstretch elasticity*, J. Mech. Mater. Struct., 11(3), 217-229 (2016)

[23] Marin, M., Baleanu, D., Carstea, C., *A uniqueness result for final boundary value problem of microstretch bodies*, J. Nonlinear Sci. Appl., 10(4), 1908-1918, (2017)

[24] Marin, M., Baleanu, D., Vlase, S., *Effect of microtemperatures for micropolar thermoelastic bodies*, Struct, Eng. Mech., 61(3), 381-387, (2017)

[25] Marin, M., Craciun, E.M., *Uniqueness results for a boundary value problem in dipolar thermoelasticity to model composire materials*, Compos. B Eng., 126, 27-37, (2017)

[26] Marin, M., Craciun, E.M., Pop N., *Some results in Green - Lindsay thermoelasticity of bodies with dipolar structure*, Mathematics, 8(4), 497, (2020)

[27] Marin, M., Codarcea, L., Chirila, A., *Qualitative results on mixed problem of micropolar bodies with microtemperatures*, Appl. Appl. Math. AAM, 12, 776–789, (2017)

[28] Marin, M., Chirila, A., Codarcea-Munteanu, L., *On a thermoelastic material having a dipolar structure and microtemperatures*, Appl. Math. Model., 80, 827–839, (2020)

[29] Marinescu, G., Tudor, C., *Sur la transformation de Laplace des distributions*, Rev. Roumanie Math. Pures Appl., 12, 1323-1327, (1967)

[30] Nieto, A.M., Rivera, J.E.M., Naso, M.G., Quintanilla, R., *Qualitative results for a mixture of Green–Lindsay thermoelastic solids*, Chaotic Mod. Simul., 3, 285–294, (2018)

[31] Pazy, P., *Semigroups od linear operators and applications to partial differential equations*, Appl. Math. Sci., Springer: New York, USA, 44, (1983)

[32] Prepeliţă, V., Pîrvan, M., Toma, A., Barbu, G., Popa, L., Roşu, D., *Transformări integrale şi funcţii complexe cu aplicaţii în tehnică*

[33] Svanadze, M., *Potential method in the theory of thermoelasticity for materials with triple voids*, Arch. Mech., 71(2), 113-136, (2019)

[34] Svanadze, M., *Uniqueness theorems in the theory of thermoelasticity for solids with double porosity*, Meccanica, 49, 2099-2108, (2014)

[35] Wang, L., *On Korn's inequality*, J. Comput. Math., 21, 321–324, (2003)

[36] Yosida, K., *Functional Analysis*, 6st ed., Springer: New York, USA, (1980)